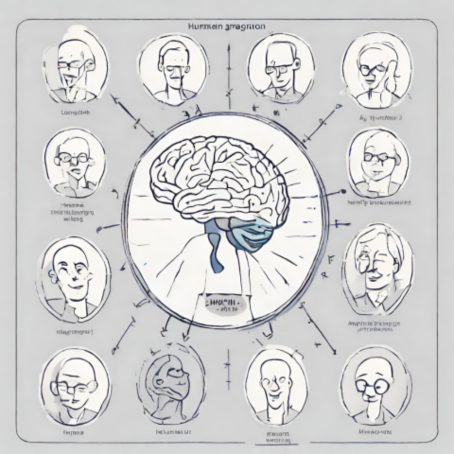
What makes us human?

What makes us human?

Thinking, reasoning, arguing, ..., and emotions



Proposition Logic

Atomic VS Compound

- Chickens are the same look;
- I will go hiking if it's sunny tomorrow;

Proposition Logic

Atomic VS Compound

- Chickens are the same look;
- I will go hiking if it's sunny tomorrow;

Grammar

$$p ::= \mathcal{A} \mid \top \mid \bot \mid \neg p \mid p_1 \wedge p_2 \mid p_1 \vee p_2 \mid p_1 \rightarrow p_2$$

It relates to boolean algebra, however boolean expressions are narrower.

There are more logic connectives (e.g. temporal logic.);
We use everyday in our languages, but we use them informally.

Modeling with Propositions

Identify atomic propositions and logic connectives

I will go hiking if it's sunny tomorrow.

Modeling with Propositions

Identify atomic propositions and logic connectives

I will go hiking if it's sunny tomorrow.

- Atomic propositions: I will go hiking (P_2); It's sunny tomorrow P_1;

Modeling with Propositions

Identify atomic propositions and logic connectives

I will go hiking if it's sunny tomorrow.

- Atomic propositions: I will go hiking (P_2); It's sunny tomorrow P_1;
- Logic connectives: if ($\rightarrow$).

Modeling with Propositions

Identify atomic propositions and logic connectives

I will go hiking if it's sunny tomorrow.

- Atomic propositions: I will go hiking (P_2); It's sunny tomorrow P_1;
- Logic connectives: if ($\rightarrow$).
- Proposition Model: $P_1 \rightarrow P_2$.

First Order Logic

Quantification: universal, existential;

Extension of proposition logic

$$p ::= ... \mid \mathcal{A}(x) \mid \forall x.p \mid \exists x.p$$

Variables in the propositions are scoped (where they are defined), the type of these variables can be implicitly inferred.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.
- Inverse and converse: $\neg P \to \neg Q \equiv Q \to P$.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.
- Inverse and converse: $\neg P \to \neg Q \equiv Q \to P$.
- Negation: $\neg(P \to Q) \equiv P \land \neg Q$.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.
- Inverse and converse: $\neg P \to \neg Q \equiv Q \to P$.
- Negation: $\neg(P \to Q) \equiv P \land \neg Q$.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.
- Inverse and converse: $\neg P \to \neg Q \equiv Q \to P$.
- Negation: $\neg(P \to Q) \equiv P \wedge \neg Q$.

Negation and quantification

- $\neg \exists x.P(x) \equiv \forall x.\neg P(x)$.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.
- Inverse and converse: $\neg P \to \neg Q \equiv Q \to P$.
- Negation: $\neg(P \to Q) \equiv P \wedge \neg Q$.

Negation and quantification

- $\neg \exists x . P(x) \equiv \forall x . \neg P(x)$.
- $\neg \forall x . P(x) \equiv \exists x . \neg P(x)$.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.
- Inverse and converse: $\neg P \to \neg Q \equiv Q \to P$.
- Negation: $\neg(P \to Q) \equiv P \land \neg Q$.

Negation and quantification

- $\neg \exists x.P(x) \equiv \forall x.\neg P(x)$.
- $\neg \forall x.P(x) \equiv \exists x.\neg P(x)$.

Equivalences (logically)

Negation and implication

- Contrapositive: $P \to Q \equiv \neg Q \to \neg P$.
- Inverse and converse: $\neg P \to \neg Q \equiv Q \to P$.
- Negation: $\neg(P \to Q) \equiv P \land \neg Q$.

Negation and quantification

- $\neg \exists x. P(x) \equiv \forall x. \neg P(x)$.
- $\neg \forall x. P(x) \equiv \exists x. \neg P(x)$.

De Morgan's Laws

- $\neg(P \land Q) \equiv \neg P \lor \neg Q$.
- $\neg(P \lor Q) \equiv \neg P \land \neg Q$.

Proof Technique

Use truth table!

P	Q	$P \to Q$	$\neg P \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Q & A